The Benefits of Being Erratic:

Correcting Errors with Noise-Enhanced Gradient Algorithms

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November 2017

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- Noise enhancement is a recently studied effect in error correction algorithms, especially low-density parity-check (LDPC) codes.
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  - Threshold sequences tailored to dominant trapping sets.

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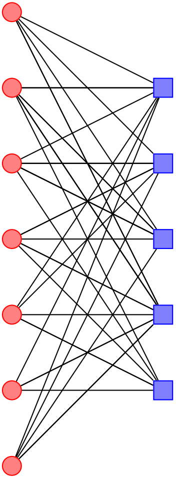

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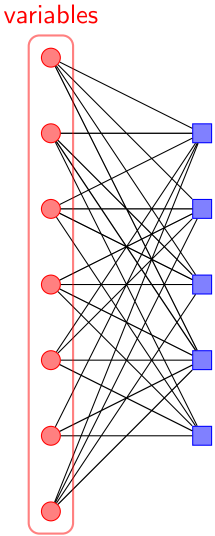

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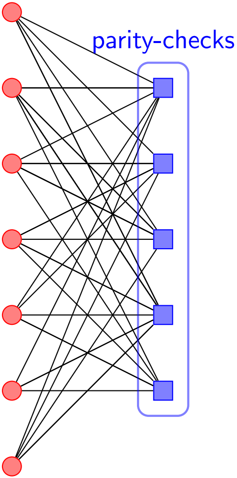

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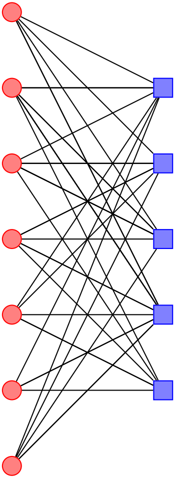

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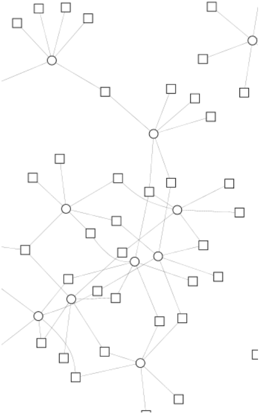
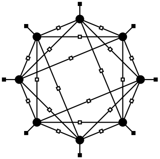
This problem is usually modeled as a Bayesian Belief Network, using the Belief Propagation (BP) algorithm. Probability calculations can be expensive, and **cycles in the network** distort the calculations, leading to failures.

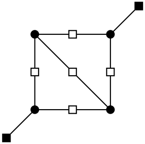


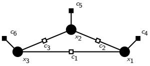
Trapping Sets

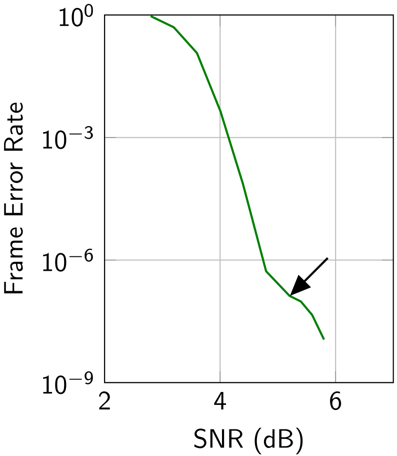
Trapping sets or Absorbing Sets are repeated cyclic patterns in the code graph.

They induce error floors in LDPC decoders based on BP.







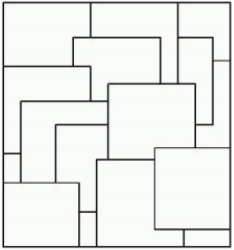


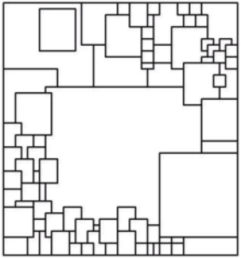
FAIDs & Decoder Diversity

There are many approaches for dealing with absorbing sets.

The general framework of Finite Alphabet Iterative Decoders (FAID) describes a class of decoders by LUTs rather than explicit Bayesian calculations[a].

Any particular FAID has small error coverage, but decoder diversity guarantees total coverage by alternating between different LUTs.





[a]Planjery et al. 2013; Declercq et al. 2013.

Single-Bit Algorithms

Several classes of low-complexity algorithms have been studied. Some notable algorithms:

- Stochastic Decoding[1] – Emulate BP using stochastic arithmetic.

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- Differential Decoding w/ Binary Message Passing (DD-BMP)[3] – Uses a memory effect inspired by relaxation algorithms.
- Bit-Flipping – Heuristics *loosely based* on BP or gradient descent.

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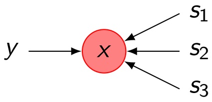
Parallel Bit-Flipping Decoders

A binary alternative, simpler than BP or Stochastic decoders.

Inputs

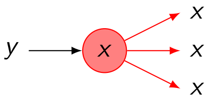
in



channel sample y_i.
hypothesis $x_i \in \{-1, +1\}$.
parity checks $s_j \in \{-1, +1\}$

Operations

out



Reliability metric: Δ_i (unique to algorithm)
Threshold update: flip x_i if $\Delta_i \leq \theta$

Then transmit x_i to adjacent nodes.

Example Algorithms

Impr. Diff. Binary (IDB):[4] $\Delta_i^{(t+1)} = \Delta_i^{(t)} + w \sum_{j \in \mathcal{M}_i \setminus j} s_{i,j} - d\, x_i^{(t)}$

Grad. Desc. BF (GDBF):[5] $\Delta_i^{(t+1)} = x_i y_i + \sum_j s_j$

Noisy GDBF:[6] $\Delta_i^{(t+1)} = x_i y_i + w \sum_j s_j + q$

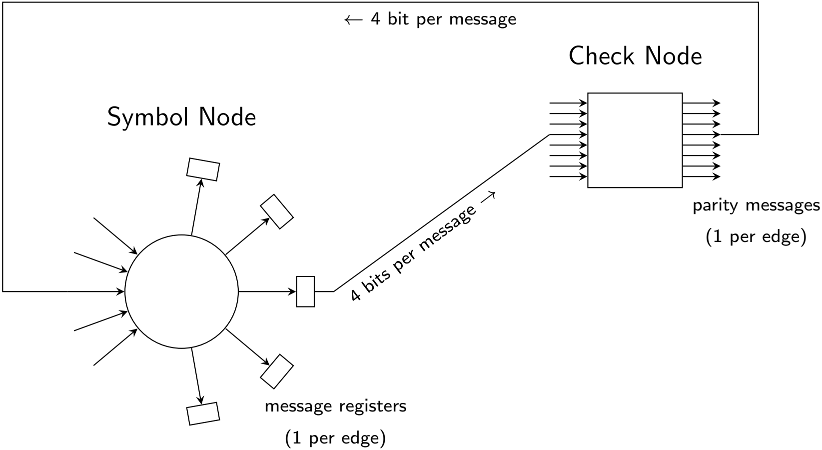
Probabilistic GDBF:[7] If $\Delta_i < \theta$, flip *with probability* p.

Traditional rule of thumb: higher complexity = better performance.

[4] Cushon et al. 2014.
[5] Wadayama et al. 2008.
[6] Sundararajan, Winstead, and Boutillon 2014.
[7] Rasheed, Ivanis, and Vasic 2014.

Motivation: Reduced Complexity

Standard BP-based or FAID algorithm:



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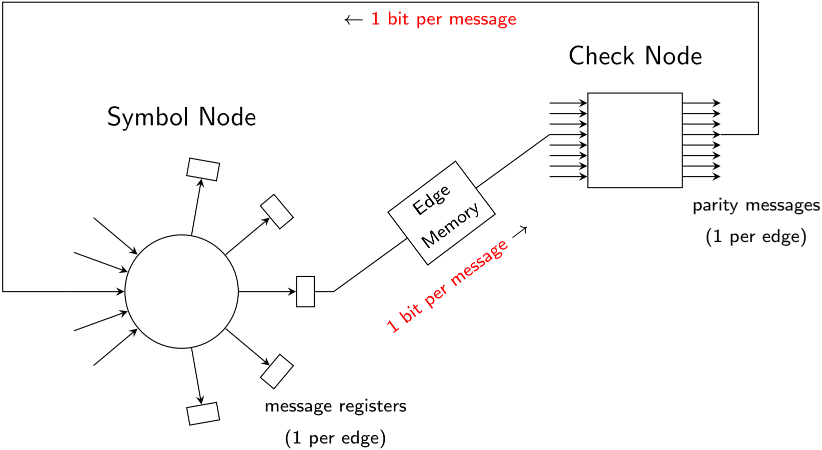
Stochastic algorithm (successive relaxation):



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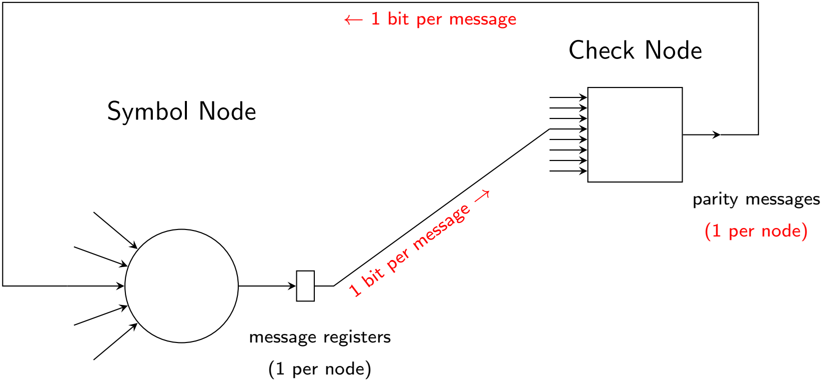
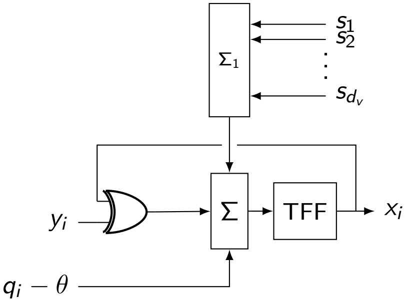
Bit-flipping algorithm:

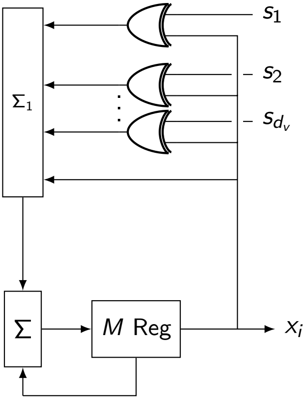


Example Node Designs

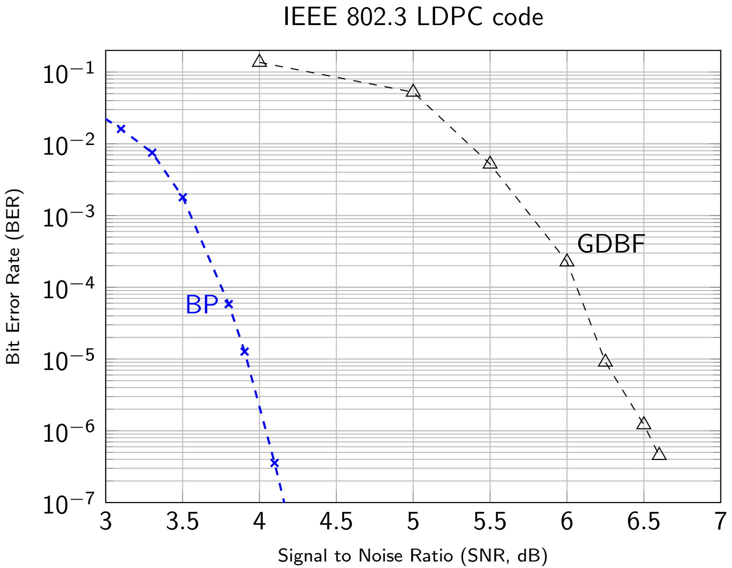
NGDBF



IDB



Performance: Noise Enhancement



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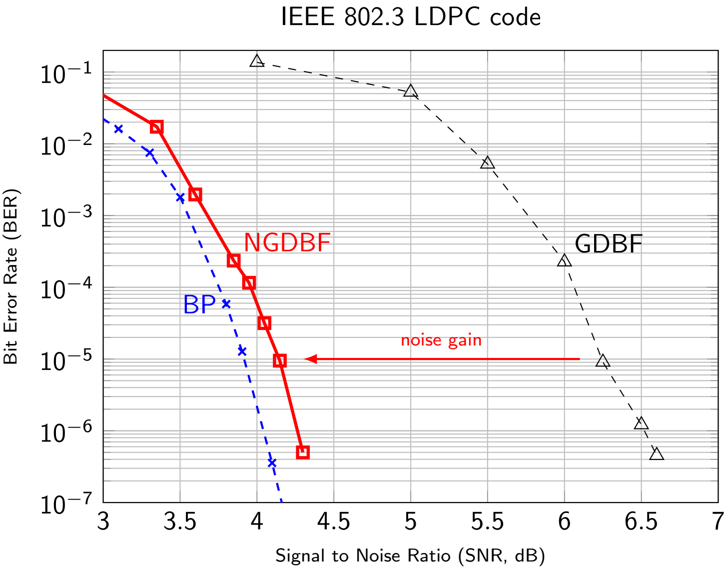
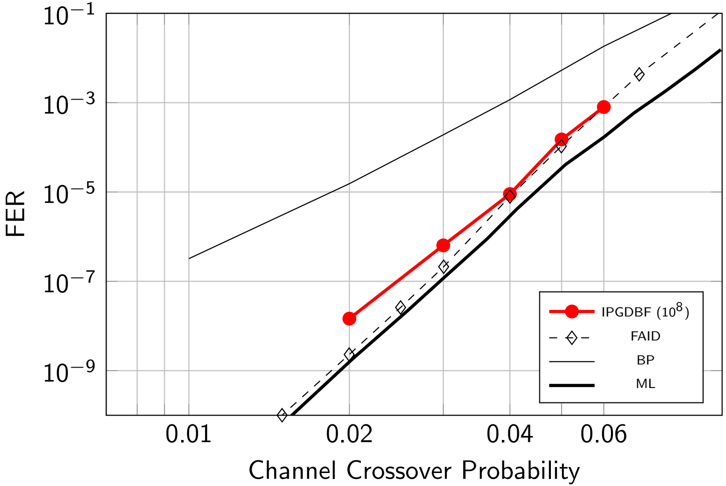


PGDBF FER

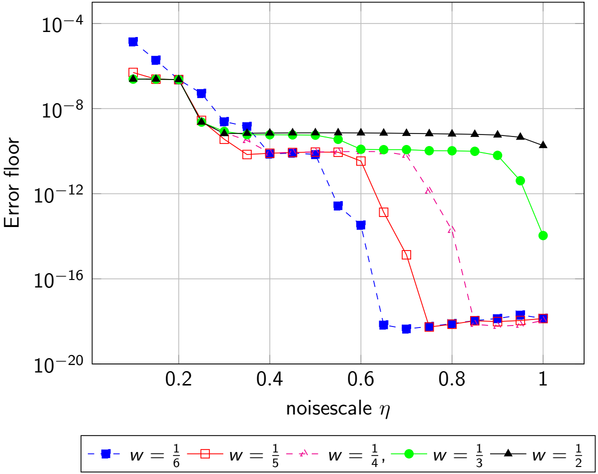
FAID and PGDBF approach Maximum Likelihood (ML) limit[8]



[8]Declercq et al. 2016.

We know that noise works...

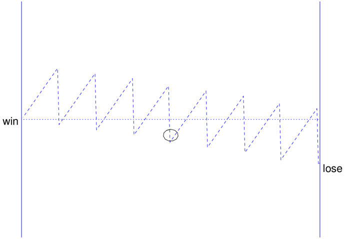
Treating each trapping set as a Markov process, we can predict the error rate and optimize algorithm parameters:

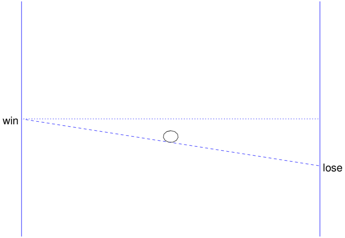


What is the bigger picture? *Why* does non-determinism enhance these algorithms?

Parrondo's "paradox"

Parrondo's paradox considers noise-perturbed particles on various sloped surfaces. These are "failing" strategies:[9]





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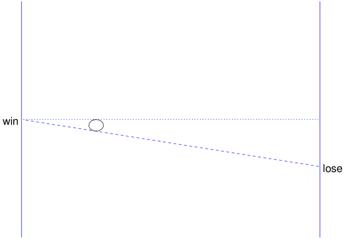
But by alternating between losing strategies at the right times, and with the right amount of noise energy, it's possible to "win" with high probability:



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- Bit-flipping algorithms are "losers," and noise turns them into "winners" . . .
- But maybe there are other ways to achieve this diversity.

Parity-Only Bit Flipping

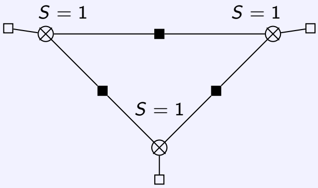
When NGDBF fails in the error floor region, it is most likely due to a single trapping set. We can try to correct it by switching strategies:

$$\text{NGDBF calculation:} \quad E_i = x_i y_i + w \sum s_{ij} + q_i$$

$$\text{New strategy:} \quad E_i = \cancel{x_i y_i} + w \sum s_{ij} + \cancel{q_i}$$

In this strategy, diversity is supplied by a dynamic threshold sequence.

Example



If all symbols are errors, a threshold of $\theta = 1$ will correct them all.

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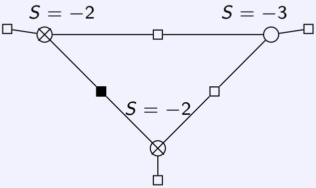

If two are errors, a threshold of $\theta = -3$ will correct two but leave one uncorrected error.

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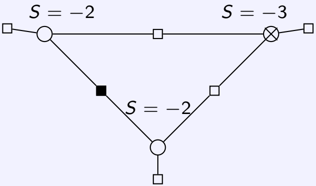

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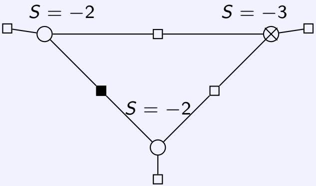
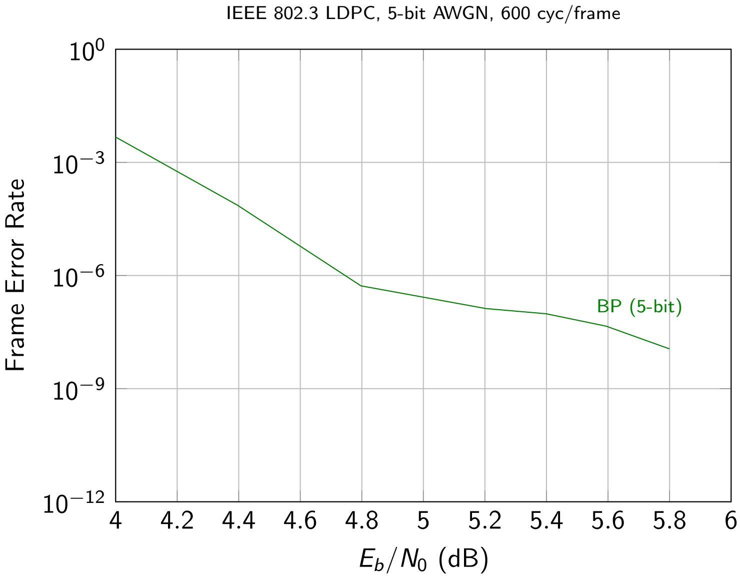


A dynamic sequence $\theta = 1, -3, -3$ covers all errors for this trapping set.

Lowering the Error Floor



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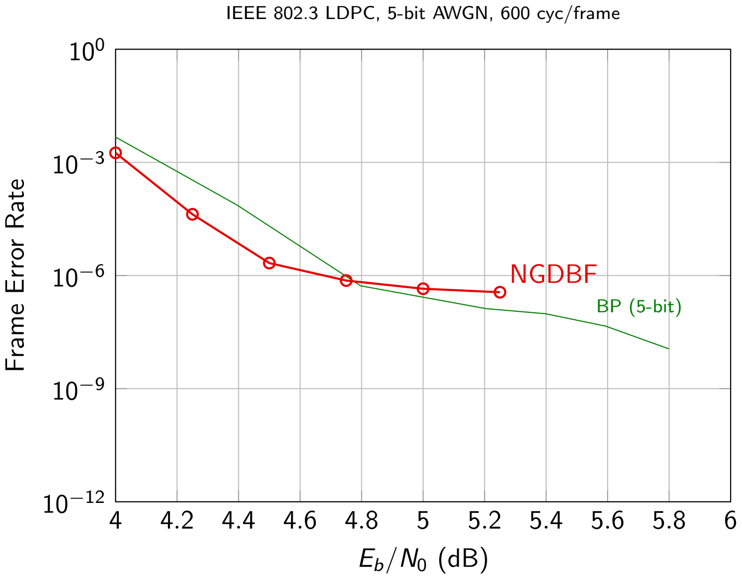

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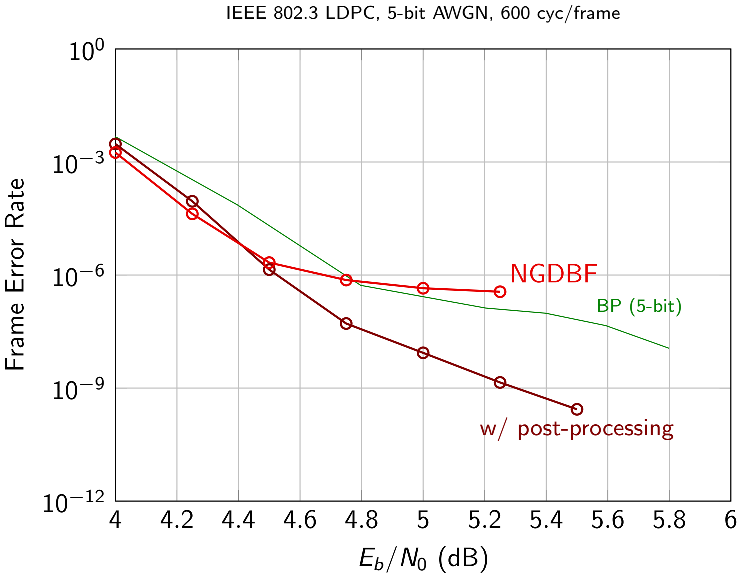

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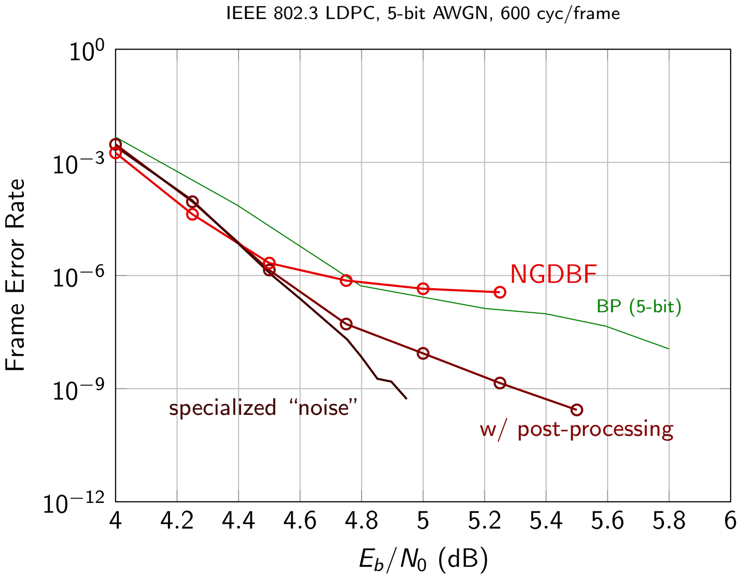

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- Each improvement simultaneously reduces implementation complexity while enhancing performance.
- We now have a basic theory to explain and optimize noise enhancement.
- Algorithm diversity remains an open frontier.

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